وإذاً:

$$f(x) = \frac{0}{2} + 0\cos x + \frac{2}{1}\sin x + 0\cos 2x + \frac{-2}{2}\sin 2x$$

$$+ 0\cos 3x + \frac{2}{3}\sin 3x + \cdots$$

$$f(x) = \frac{2}{1}\sin x - \frac{2}{2}\sin 2x + \frac{2}{3}\sin 3x + \cdots$$

المحاضرة (11)

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos \omega_0 k x + b_k \sin(\omega_0 k x)\right)$$

$$a_0 = \frac{2}{T_0}\int_{T_0} f(x)\,dx$$

$$a_k = \frac{2}{T_0}\int_{T_0} f(x)\cdot\cos(\omega_0 k x)\,dx$$

$$b_k = \frac{2}{T_0}\int_{T_0} f(x)\sin(\omega_0 k x)\,dx$$

مثال: أوجد نشر فورييه المثلثي للدالة الدورية

$$f(x) = \begin{cases} -1 & 0 \le x < \pi \\ 1 & \pi \le x < 2\pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\left(\int_0^{\pi} -1\,dx + \int_{\pi}^{2\pi} 1\,dx\right)$$

$$= \frac{1}{\pi}\left(-x\Big|_0^{\pi} + x\Big|_{\pi}^{2\pi}\right) = \frac{1}{\pi}\left(-\pi - 0 + 2\pi - \pi\right)$$

$$= 0$$

لأن $\sin \pi K = 0$

$\sin 2\pi K = \sin 0 = 0$

$$a_K = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos Kx\,dx$$

$$= \frac{1}{\pi}\left(\int -\cos Kx\,dx + \int_{\pi}^{2\pi} \cos Kx\,dx\right)$$

$$= \frac{1}{\pi}\left(-\frac{\sin Kx}{K}\Big|_0^{\pi} + \frac{\sin Kx}{K}\Big|_{\pi}^{2\pi}\right)$$

$$= \frac{1}{\pi}\left(-\frac{\sin K\pi}{K} + 0 + \frac{\sin 2\pi K}{K} - \frac{\sin K\pi}{K}\right)$$

$$\Rightarrow a_K = \frac{1}{\pi}(0+0+0-0) = 0$$

لأن $\begin{cases} \sin 2\pi K = \sin 0 = 0 \\ \sin K\pi = 0 \end{cases}$

$$b_K = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin Kx\,dx$$

$$= \frac{1}{\pi}\left(\int_0^{\pi} -\sin Kx + \int_{\pi}^{2\pi} \sin Kx\,dx\right)$$

$$= \frac{1}{\pi}\left(+\frac{\cos Kx}{K}\Big|_0^{\pi} - \frac{\cos Kx}{K}\Big|_{\pi}^{2\pi}\right)$$

$$b_K = \frac{1}{\pi}\left(\frac{\cos K\pi}{K} - \frac{1}{K} - \left(\frac{\cos 2\pi K}{K} - \frac{\cos K\pi}{K}\right)\right)$$

$$= \frac{1}{\pi K}(2\cos K\pi - 1 - 1)$$

لأن $\cos 2\pi K = \cos 0 = 1$

$$= \frac{1}{\pi K}(2\cos K\pi - 2)$$

$$= \frac{2}{\pi K}(\cos K\pi - 1) = \begin{cases} \frac{2}{\pi K}(1-1) = 0 & \text{زوجي } K \\ \frac{2}{\pi K}(-1-1) = \frac{-4}{\pi K} & \text{فردي } K \end{cases}$$

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وضعه نشر فوريه للدالة:

$$f(x) = 0 + \frac{-4}{\pi(1)} \sin x + \left(\frac{-4}{\pi(3)}\right) \sin 3x + \dots$$

مثال: اكتب نشر فوريه المثالي للدالة الدورية $f(x)$ والتي دورها $T_0 = 2\pi$ والمعرفة بالشكل:

$$f(x) = \begin{cases} -x & -\pi \le x < 0 \\ x & 0 \le x \le \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\left(\int_{-\pi}^{0} -x\,dx + \int_{0}^{\pi} x\,dx\right)$$

$$= \frac{1}{\pi}\left(-\frac{x^2}{2}\Big|_{-\pi}^{0} + \frac{x^2}{2}\Big|_{0}^{\pi}\right)$$

$$= \frac{1}{\pi}\left(0 + \frac{\pi^2}{2} + \frac{\pi^2}{2} - 0\right)$$

$$= \pi$$

لحساب a_K

$$a_K = \frac{1}{\pi}\left(\int_{-\pi}^{0} \underset{u}{-x}\, \underset{dv}{\cos Kx\,dx} + \int_{0}^{\pi} \underset{u}{x}\, \underset{dv}{\cos Kx\,dx}\right)$$

du

$$a_K = \frac{1}{\pi}\left(-\left(x\frac{\sin Kx}{K}\Big|_{-\pi}^{0} - \int_{-\pi}^{0} \frac{\sin Kx}{K}\,dx\right) + \left(x\frac{\sin Kx}{K}\Big|_{0}^{\pi} - \int_{0}^{\pi} \frac{\sin Kx}{K}\,dx\right)\right)$$

$$= \frac{1}{\pi}\left(\left(-0+(-\pi)\frac{\sin(-K\pi)}{K}+\frac{1}{K^2}(\cos Kx)\Big|_{-\pi}^{0}\right)\right.$$

$$\left.+\left(\pi\frac{\sin K\pi}{K}-0+\frac{1}{K^2}\cos Kx\Big|_{0}^{\pi}\right)\right)$$

$$= \frac{1}{K^2\pi}\left(-(1-\cos K\pi)+\cos\pi K-1\right) \Rightarrow$$

$$= \frac{1}{\pi K^2}\left(-2+2\cos K\pi\right) = \frac{2}{\pi K^2}\left(-1+\cos\pi K\right)$$

$$= \begin{cases} 0 & \text{، } K \text{ زوجي} \\ -\dfrac{4}{\pi K^2} & \text{، } K \text{ فردي} \end{cases}$$

The End